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$$a_{r-1\alpha}\, x_\alpha + l_{r-1} + k_{r-1} = 0,$$
$$a_{r\alpha}\, x_\alpha + \phi_0(l_1, l_2, \ldots, l_{r-1}) + k_r = 0,$$

. . . .

. . . .

$$a_{m\alpha}\, x_\alpha + \phi_{m-r}(l_1, l_2, \ldots, l_{r-1}) + k_m = 0,$$

where $\phi_0, \ldots, \phi_{m-2}$ are the linear functions of $l_1, l_2, \ldots, l_{r-1}$.

The system T' is consistent since T is consistent and it must be of rank r since T has this rank. Hence it results that one and only one set of values for x_α, $l_1, \ldots, l_{r-1}$ will satisfy the first r equations. That is, x_α can have only one value in system T'. It can therefore have only one value in system T and the theorem in question has been established. From this theorem it results that each unknown in system T has either only one value or it has an infinite number of values whenever this system is consistent. In particular, a system of linear equations has always an infinite number of distinct solutions whenever it has more than one solution.

From the above it is evident that the language as regards solvability of a system of linear equations becomes much more concise by means of the concept of rank. Although this concept is comparatively new in mathematics it is of such fundamental importance that it should occupy a more prominent place in the courses in advanced algebra. It should be remembered that a thing can only appear simple when we can see clearly through it. In particular, the theory of linear equations appears simple only after an exhaustive study by means of such a powerful instrument as the concept of rank.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

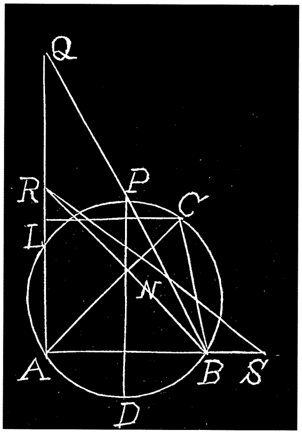
GEOMETRY.

363. Proposed by G. I. HOPKINS, Manchester, N. H.

Construct the triangle, having given, base, vertical angle, and difference between altitude and sum of the other two sides.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In what follows we assume that $AB=a=$the given base; $\angle ACB=\angle C=$given vertical angle; $p=$difference of altitude and sum of other sides.



We further assume that $p > a < a\cot\frac{1}{2}C$. The solution would be just as simple with any other possible assumption.

On AB describe the segment ACB containing the given angle C. Draw the diameter PD perpendicular to AB. Also draw AQ perpendicular to AB. Draw BP meeting AQ in Q. With B as a center and a radius equal to p, describe an arc cutting AQ in R. With R as a center and a radius equal to $AQ-p$, describe an arc cutting AB produced in S. On SR measure off $SN=SA$. Then RN=altitude required. Take $AL=RN$, and draw CL parallel to AB, cutting the circle in C. Draw AC, BC. Then ACB is the required triangle. Let x, y, z be the sides BC, AC, and the altitude. Then $xy\sin C=az$, $x+y-z=p$, $a^2=x^2+y^2-2xy\cos C$.

$$\therefore a^2+2xy(1+\cos C)=(p+z)^2.$$

$$\therefore z^2-2(a\cot\tfrac{1}{2}C-p)z=a^2-p^2.$$

$$\therefore z=a\cot\tfrac{1}{2}C-p-\sqrt{[(a\cot\tfrac{1}{2}C-p)^2-(p^2-a^2)]}.$$

$\angle AQB=\frac{1}{2}C$, $AQ=a\cot\frac{1}{2}C$, $RS=a\cot\frac{1}{2}C-p$, $AR=\sqrt{[p^2-a^2]}$, $AS=\sqrt{[(a\cot\frac{1}{2}C-p)^2-(p^2-a^2)]}$.

$\therefore RN=z$. $\therefore$ The triangle ACB contains all the required parts.

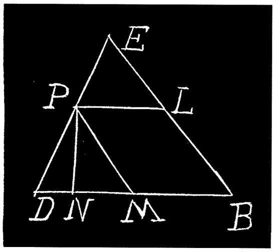
Also solved by J. Scheffer.

364. Proposed by R. C. ARCHIBALD, Providence, R. I.

Between the side of a given rhombus and its adjacent side produced, to insert a straight line of a given length and directed to the opposite corner. ["Euclidean constructions" are particularly desired.]

Solution by C. N. SCHMALL, New York City, and J. SCHEFFER, A. M., Hagerstown, Md.

This construction cannot be effected by Euclidean geometry. This will be evident from the algebraic analysis of the conditions.



Let $MBLP$ be the given rhombus, and P the given corner. Also let DE be the required line, so that $DE=l$=given length. On AB drop the perpendicular PN. Now let $MB=BL=a$, $MN=c$, $DM=x$. Then we have, in $\triangle DMP$,

$$DP^2=DM^2+MP^2-2DM.MN=x^2+a^2-2cx.$$

Also, $DM^2:DP^2=DB^2:DE^2$;